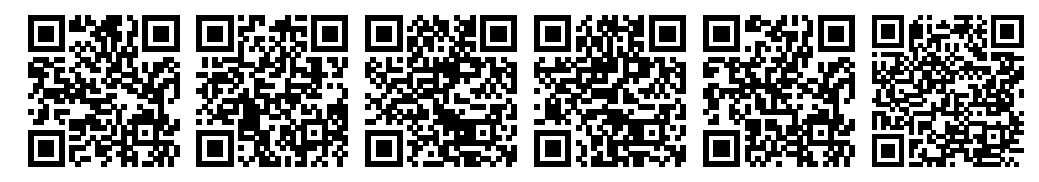


Advanced matrix and tensor methods for interpolative decomposition, Markov chain compression, and chemical design

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1 Introduction Emerging scientific computing in modeling, simulation, and artificial intelligence demands new and advanced methods for discrete algorithms.



2 A new approach and theory for the column subset selection problem Lorem ipsum dolor sit amet, consectetur adipiscing elit. Etiam lobortis facilis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit. Ut porttitor. Praesent in sapien. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Duis fringilla tristique neque. Sed interdum libero ut metus. Pellentesque placerat. Nam rutrum augue a leo. Morbi sed elit sit amet ante lobortis sollicitudin. Praesent blandit blandit mauris. Praesent lectus tellus, aliquet aliquam, luctus a, egestas a, turpis. Mauris lacinia lorem sit amet ipsum. Nunc quis urna dictum turpis accumsan semper.

Consider the following linear system:

$$Ax = b, \quad (1)$$

where $A \in \mathbb{R}^{m \times n}$ is sparse and $b \in \mathbb{R}^m$.

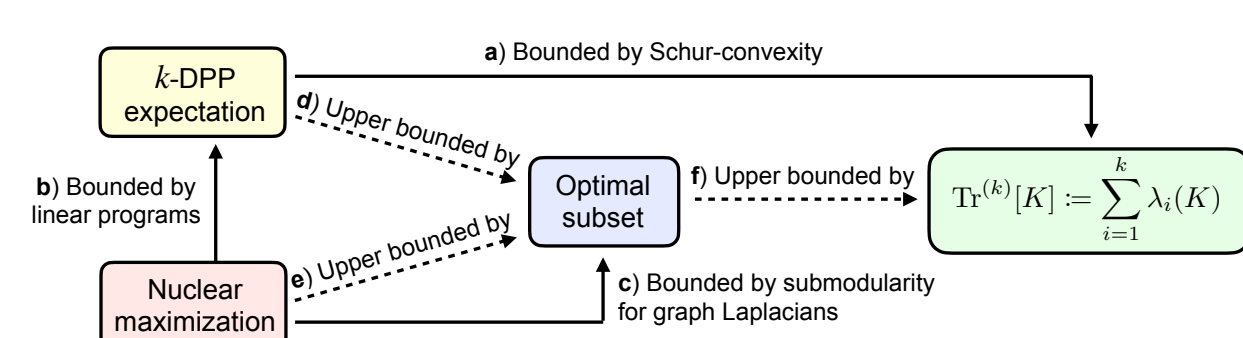


Figure 1: Theoretical error bounds.

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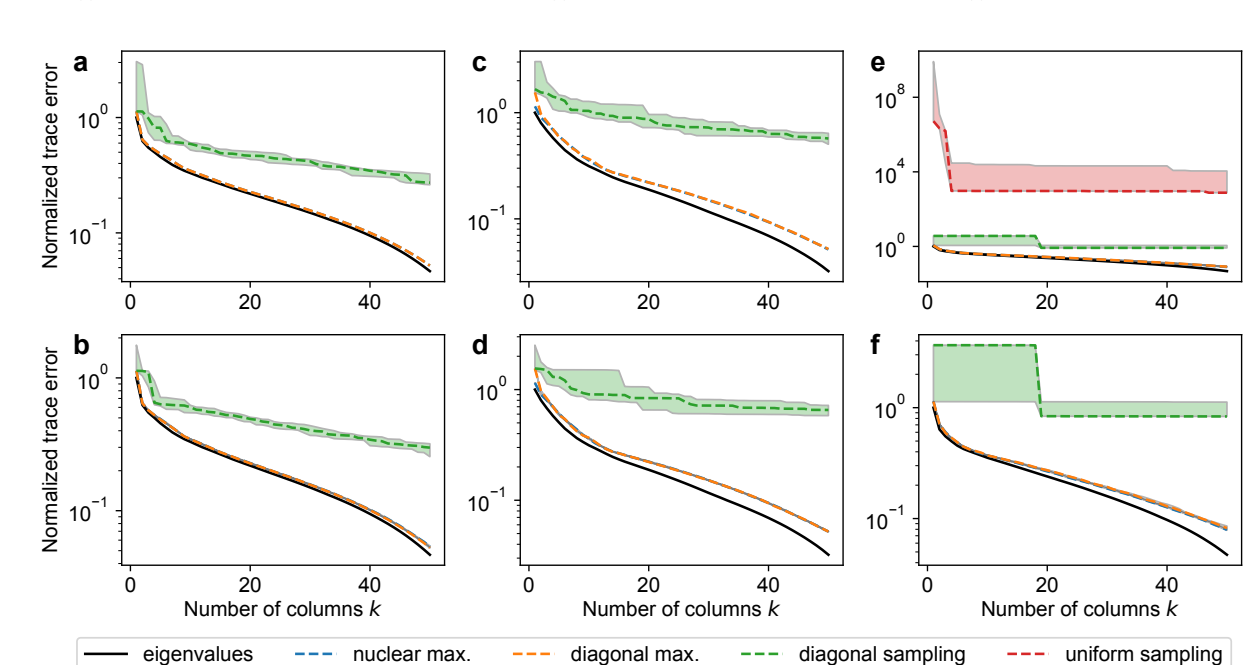


Figure 2: RNA Laplacian CSSP.

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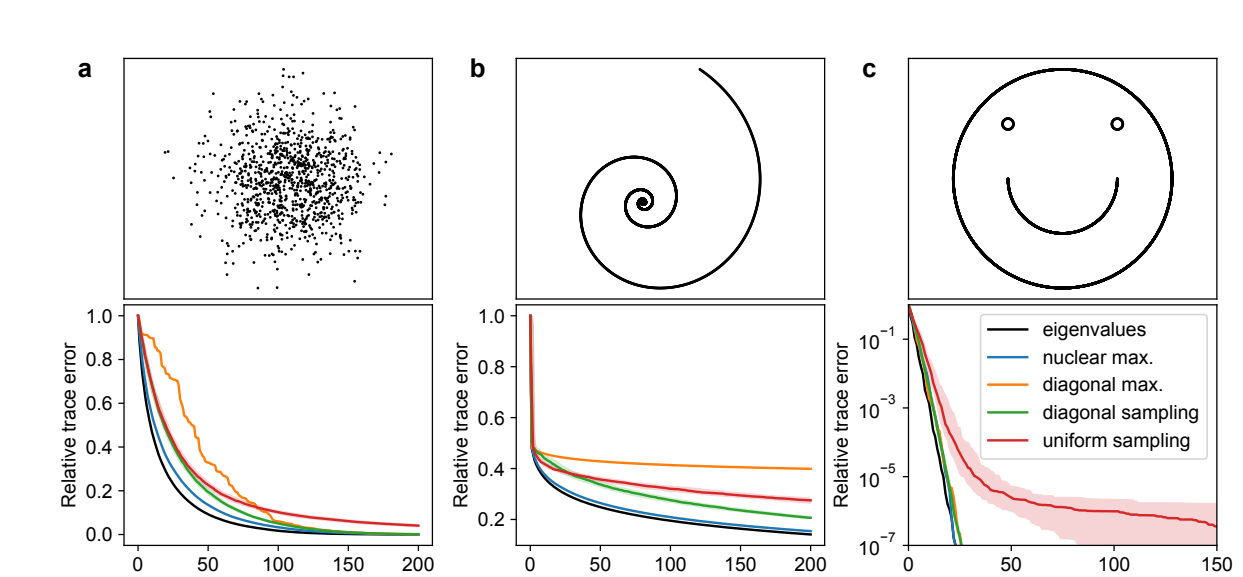


Figure 3: CSSP examples.

3 Approximating tensors with accurate and scalable interpolative decomposition

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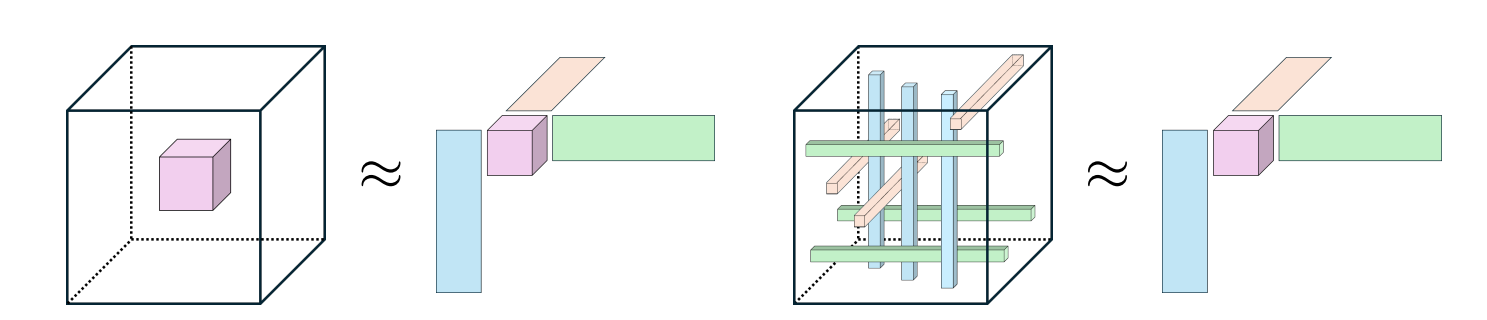


Figure 4: Tensor compression via interpolative decomposition

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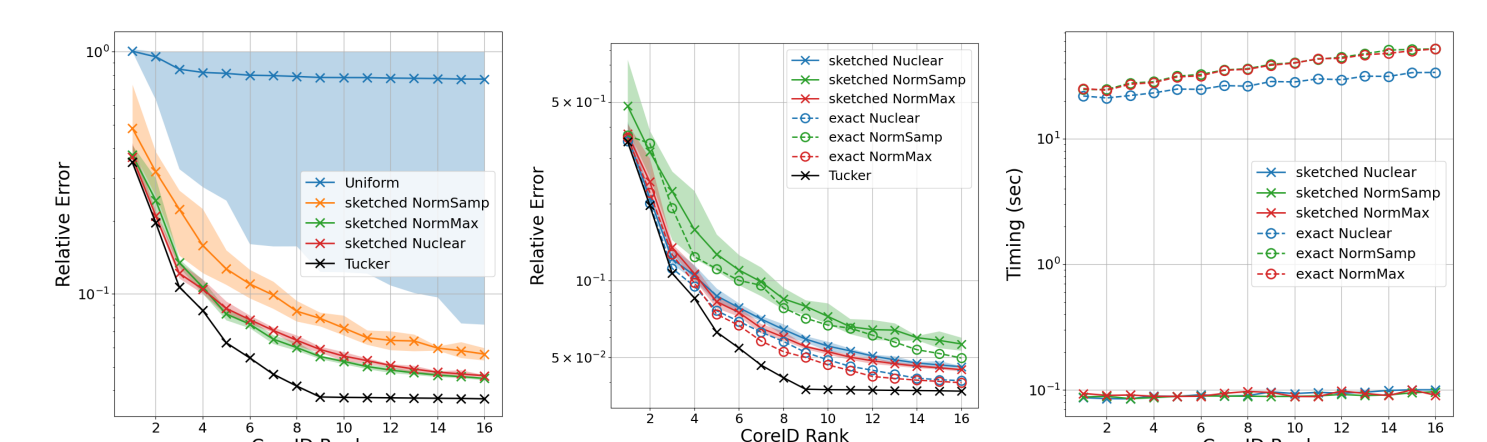


Figure 5: **Left:** Reconstruction error of the CP CoreID algorithm with sketching, including the average error (solid lines) and min-max errors (shaded region) for 4 matrix ID algorithms. On the bottom, the Tucker error is plotted as the optimal low rank approximation error. **Middle:** Comparison of average reconstruction error for sketched and exact algorithms. **Right:** Average computation times for sketched and exact algorithms.

4 New algorithms and fundamental bounds for Markov chain compression

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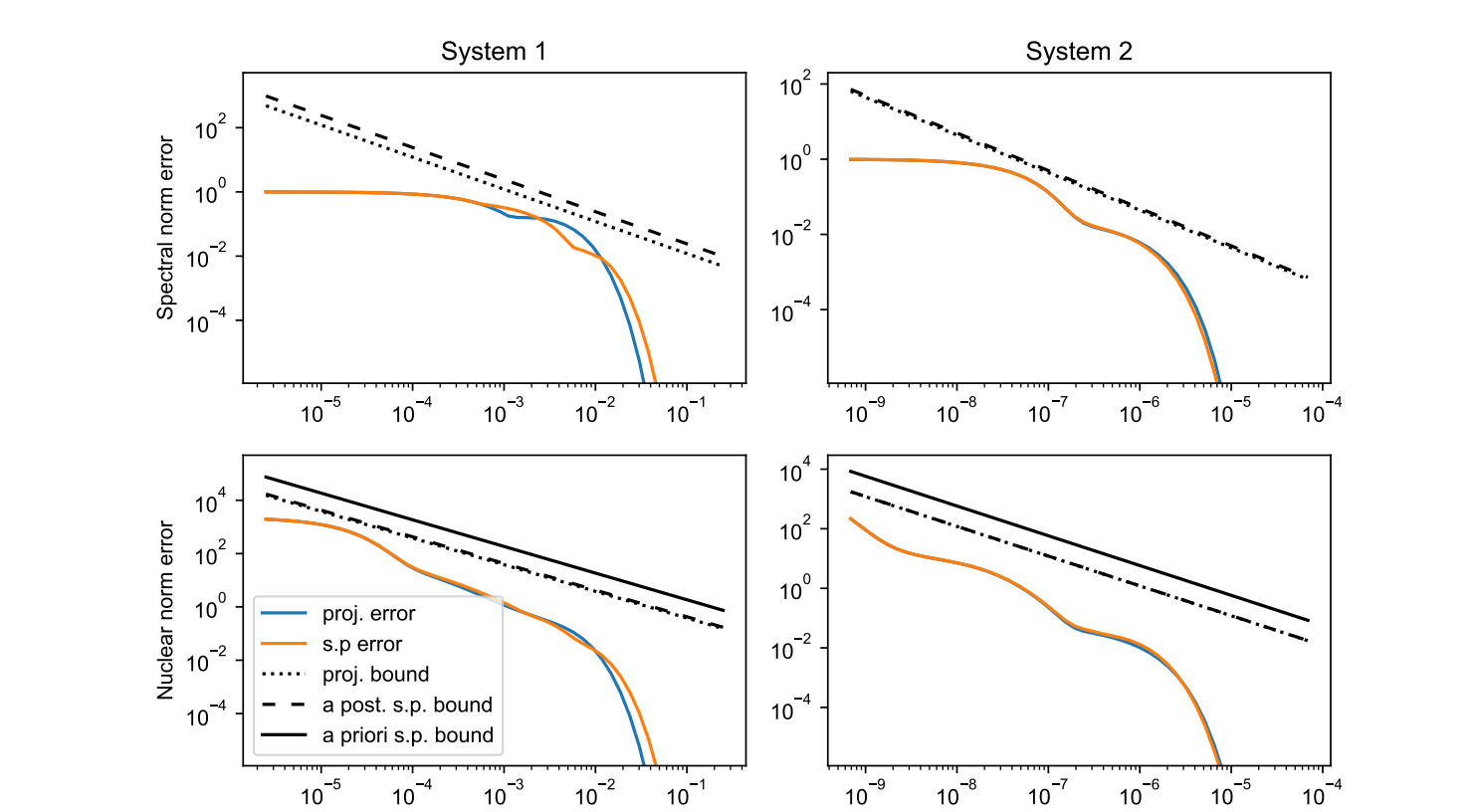


Figure 6: Example figure showing key results. Axes represent relevant quantities.

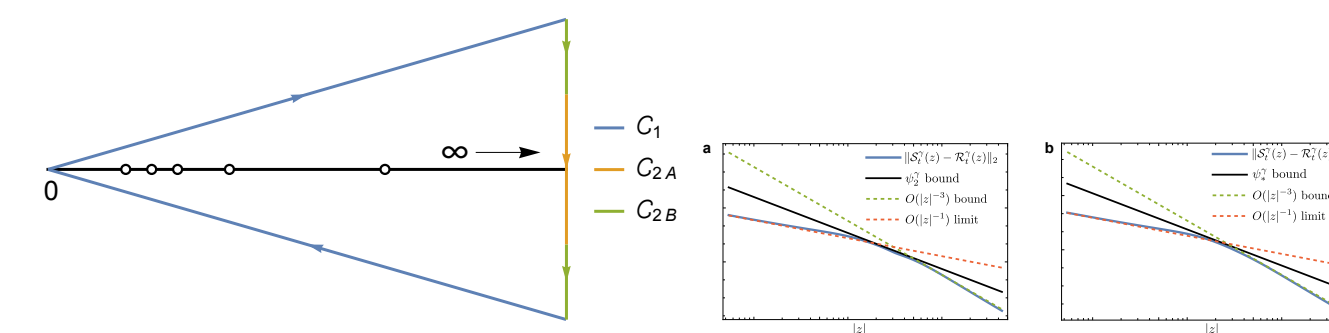


Figure 7: Cauchy integral-based contour integration bound.

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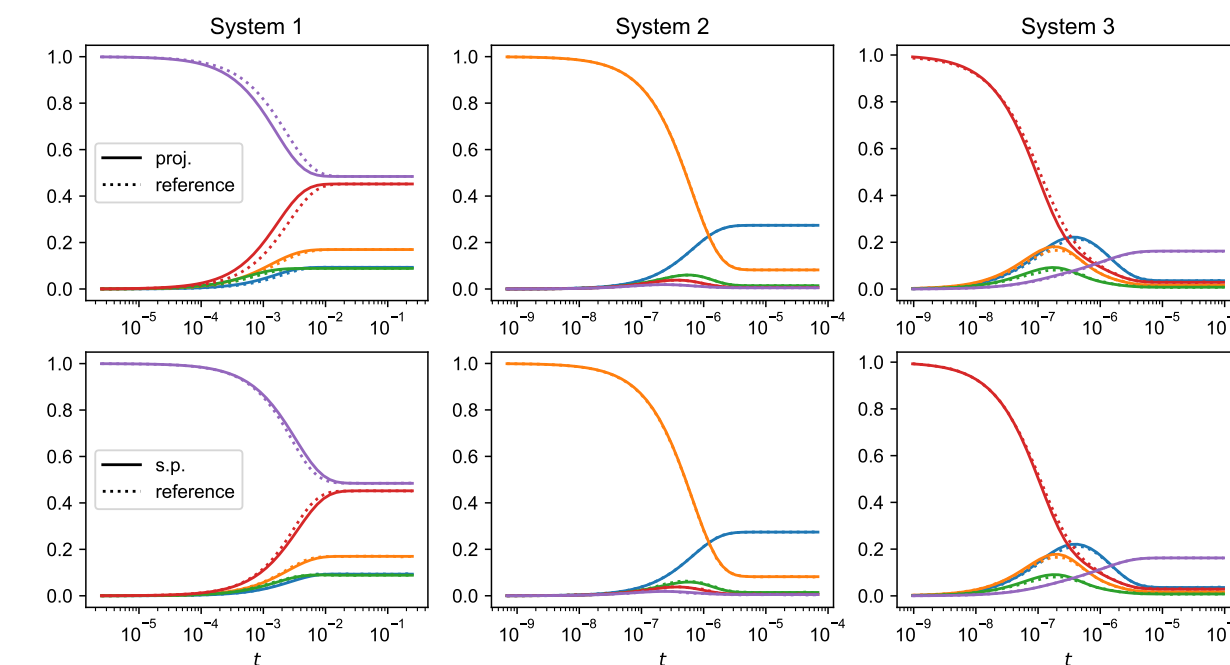


Figure 8: Compression of Markov chain dynamics.

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5 Tensor-based analysis and design of nucleic acid secondary structures

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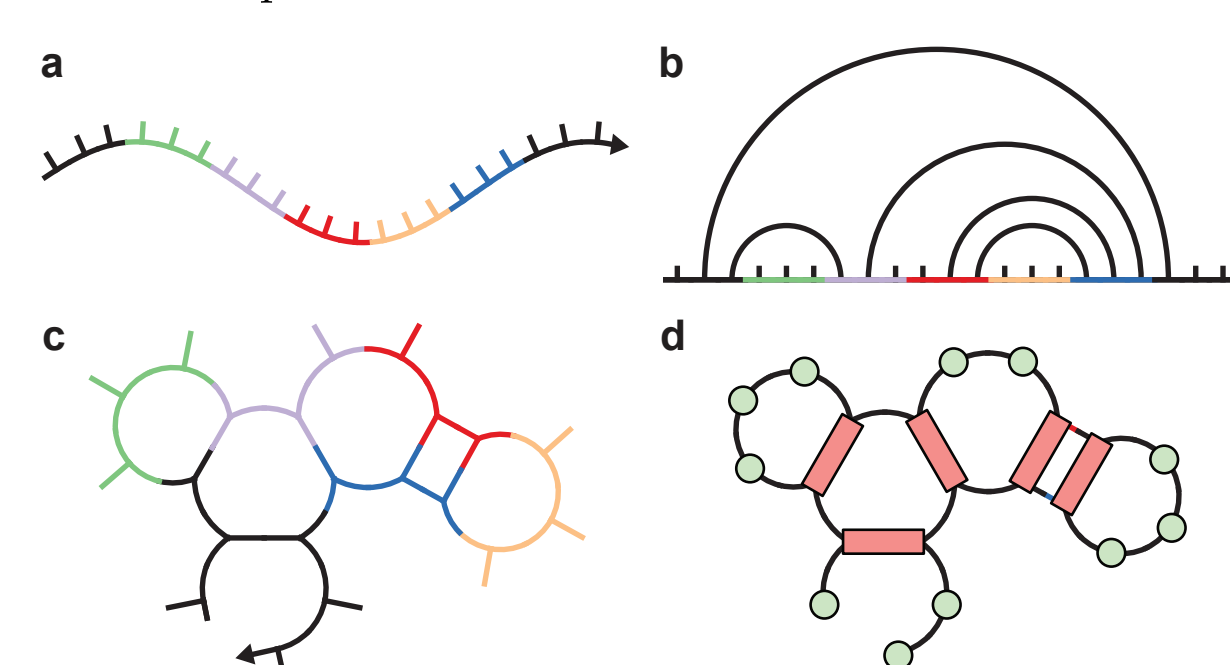


Figure 9: Example figure showing key results. Axes represent relevant quantities.

5.1 First Method Lorem ipsum dolor sit amet, consectetur adipiscing elit. Etiam lobortis facilis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit.

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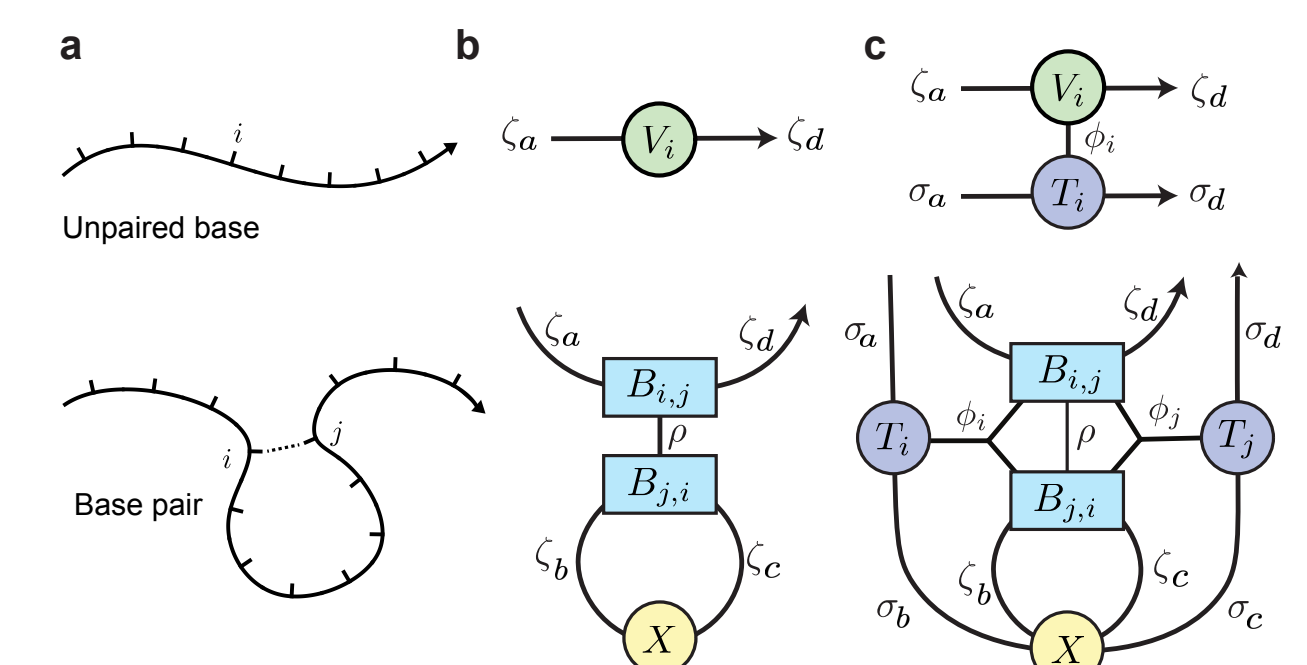


Figure 10: Example figure showing key results. Axes represent relevant quantities.

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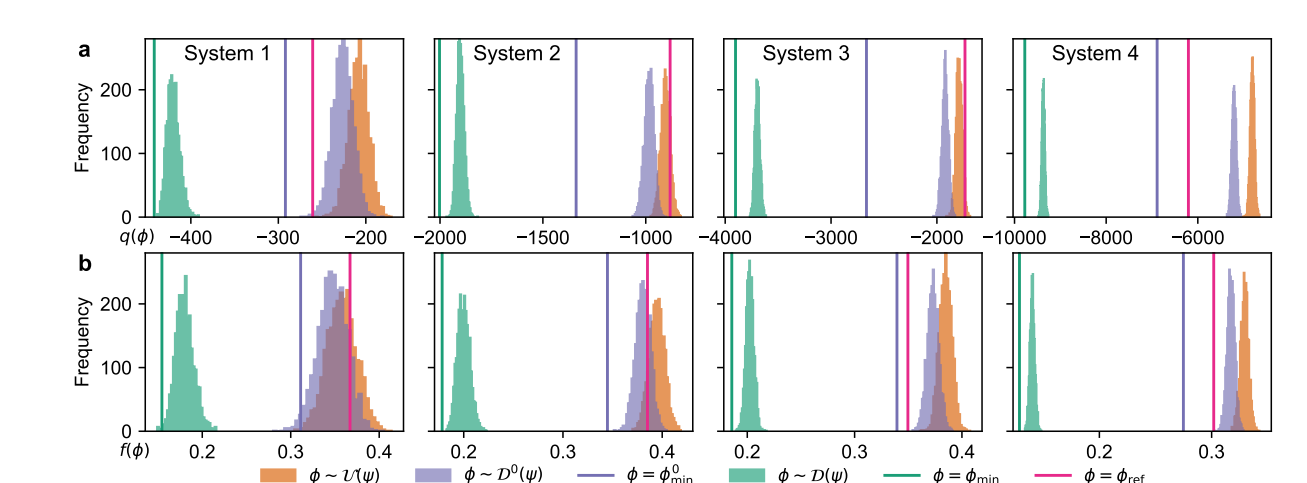


Figure 11: Example figure showing key results. Axes represent relevant quantities.

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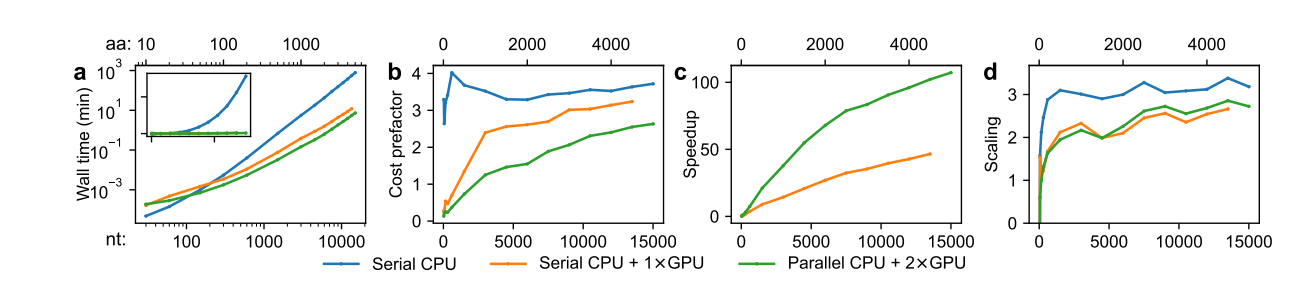


Figure 12: Example figure showing key results. Axes represent relevant quantities.

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